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Collnet Journal of Scientometrics and Information Management

ISSN: 0973-7766 (Print) 2168-930X (Online) Journal homepage: http://www.tandfonline.com/loi/tsim20

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To cite this article: Constantinos T. Artikis & Panagiotis T. Artikis (2009) Learning Processes and Information Management Operations Utilising a Class of Random Sums, Collnet Journal of Scientometrics and Information Management, 3:1, 31-38, DOI: 10.1080/09737766.2009.10700862

To link to this article: http://dx.doi.org/10.1080/09737766.2009.10700862


Published online: 04 Jun 2013.


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Learning Processes and Information Management Operations Utilising a Class of Random Sums

Constantinos T. Artikis
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The present paper concentrates on a geometric random sum of nonnegative, independent and identically distributed random variables. A stochastic derivation of such a random sum is established. Moreover, the paper provides an interpretation of the nonnegative geometric random sum as a constituent factor of learning processes in evaluating the performance of information risk control operations.

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1. Introduction

Scientists make use of models in order to describe, clarify, predict or assess aspects of the complex reality. Beyond the extremely important role of models in the scientific approach, there seems to be a very strong connection between our mental processes and these structures that outline, interpret and subsequently direct our ideas. Models capture the nature of human understanding and can consequently be considered as a valuable element of cognitive processes. The modelling process includes steps of model formulation, model validation and model application. Learning is a process of modelling where a very wide variety of models are formulated for creating an understanding of reality. (Wahlström [1])

Models are useful for understanding the causal procedures of a real system. Such an understanding makes it easier to choose inflows for affecting outflows in a beneficial direction. Models are also useful for making predictions of future behaviour of real systems. The predictions make it viable to efficiently choose actions which give desired results. In this way models are used as instruments for optimal control plans. Models can also be useful for decision makers training. There is frequently a need to incorporate many models into a consolidated structure to provide users with different views of the system. (Wahlström [1]) A fundamental category of models, with substantial contribution to the investigation of systems arising in many different practical disciplines, is the category of stochastic models. It is generally recognized that random sums constitute a class of stochastic models with interesting applications in many practical disciplines. This recognition is based on the incorporation of two powerful concepts of probability theory in the mathematical structure of a random sum. The first concept is a sequence of independent and identically distributed random variables and the second concept is a discrete random variable taking values

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in the set of nonnegative integers and being independent of the sequence. These probabilistic concepts make the mathematical structure of a random sum suitable for solving problems in a wide variety of practical disciplines (Tijms [2]). It is known that the applicability of random sums is strongly supported by the powerful results of the theory of characteristic functions (Lukacs [3]), moreover, the use of Fast Fourier Transform substantially facilitates the applications of random sums in insurance, risk management and operational research (Paulson and Dixit [4]). Renewal theory, waiting time problems and stochastic processes make extensive use of random sums. The most noteworthy special case is that of the Poisson random sum. The characteristic function of a Poisson random sum belongs to the family of infinitely divisible characteristic functions. Characteristic functions of Poisson random sums play a very important role in the study of infinitely divisible distributions. The limit of a sequence of characteristic functions belonging to Poisson random sums is infinitely divisible. The converse is also true. Every infinitely divisible characteristic function can be written as the limit of a sequence of characteristic functions belonging to Poisson random sums (Lukacs [3]). The most significant practical consideration of infinitely divisible distributions is in stochastic modelling activities. Several stochastic models of particular practical interest can only be defined in terms of infinite divisible distributions. Another basic reason for undertaking research activities on the fundamental property of infinitely divisibility is a the fact that infinitely divisible distributions in statistics and other areas of applied probability seems to be the rule than the exception. It is easily understood that the establishment of the property of infinite divisibility for a given distribution is very useful, because this establishment implies other special properties of this distribution, such a typical tail behaviour (Steutel [5]). Since Poisson random sums constitute a particular class of stochastic models with great practical importance in management and systemics, characteristic functions of Poisson random sums are very strong analytical tools for investigating the structure of infinitely divisible distributions, some stochastic models can only be based on infinitely divisible distributions and learning is a process of modelling, then it seems to be of some practical importance to consider the role in learning processes and information management operations of geometric random sums which are special cases of Poisson random sums. Such a consideration can improve the proactive character of risk management in treatment of major risks faced by information systems supporting the evolution of modern complex organizations. This means the recognition and acceptance of information risk management as a fundamental factor of information management. The present paper mainly concentrates on the applicability in the discipline of information risk management of a nonnegative geometric random sum, which is a powerful probabilistic concept (Keilson and Steutel [6]).

Risk management has become particularly valuable with the development of modern and possibly hazardous activities. Moreover, risk management makes extensive use of stochastic models. If such models are valid descriptions of reality, they can be applied to predict and therefore also reduce the adverse effects of risks. If a stochastic model in the risk management process is applied outside its domain of validity it can introduce severe defects in risk treatment operations. The application of inadequate stochastic models creates the most crucial threat to the fundamental risk management operations. Stochastic models in risk management are based on many postulations which have to be examined if invalid explanations are to be averted. An explicit consideration of risk management is based on the fundamental matter of decision making under conditions of uncertainty which has usually been undertaken by the theory of expected utility. This theory is, however, too limited to face real situations but it can still offer significant perceptions by concentrating on the validity of essential postulations. Decision making is based on stochastic models to predict results of given choices and stochastic models are applied to outline the decision making itself. Uncertainty means that results can be predicted by making use of probability distributions(Wahlström [1]).

The present paper provides a consideration of a strong analytical tool of probability theory as an efficient stochastic model of information risk management, which is a significant component of modern risk management (Wright [7]). More precisely, it is shown that a geometric random sum of nonnegative, independent and identically distributed random variables can be interpreted as an important factor of learning

processes for supporting information risk control operations. It is also shown that nonnegative geometric random sums have to be thoroughly investigated as very strong tools of various fundamental learning processes being particularly suitable for developing, analyzing, selecting and implementing efficient control operations in the important discipline of information risk management.

The present paper is devoted to the implementation of three purposes. The consideration of a random sum based on a geometrically distributed random variable and a sequence of nonnegative, independent and identically distributed random variables constitutes the first purpose of the paper. The establishment of a stochastic derivation of such a random sum, by making use of a homogeneous Poisson process and an exponentially distributed random variable, is the second purpose of the paper. Moreover, the third purpose of the paper consists of the interpretation of the proposed geometric random sum as a fundamental factor of learning processes in evaluating the performance of control operations applied to information risks.

It is quite clear that the research purposes of the present paper can contribute to the consideration of the class of random sums of nonnegative, independent and identically distributed random variables as a useful analytical tool for various areas of risk management. Such a consideration may provide risks experts with valuable information for describing, analyzing and implementing fundamental risk management operations (Artikis and Artikis[8], Dickinson [9]).

The implementation of these purposes substantially facilitates the use of basic probabilistic concepts and significant stochastic models for establishing useful interconnections between the discipline of risk management and information systems supporting the expansion of modern complex organizations. In addition, such interconnections can be of some particular theoretical and practical importance for the evolution of information risk management as a fundamental organizational discipline. It is also shown that the implementation of the research purposes of the present paper makes very clear that the family of infinitely divisible random variables, taking values in the set of nonnegative real numbers, can contribute to the use of fundamental concepts, principles, methods and strategies from the area of proactive risk treatment to information management operations. Infinite divisibility is identified as a structural property of certain learning processes and information management operations.

2. Consideration of a Nonnegative Geometric Random Sum

The purpose of the present section of the paper is the consideration of a geometric random sum of nonnegative random variables.
We suppose that

$$\{X_n,\ n=1,\ 2,...\}$$

is a sequence of nonnegative and independent random variables distributed as the random variable

$$X$$

with characteristic function

$$\varphi_X(u)$$

.

We also suppose that

$$N$$

is a discrete random variable independent of the above sequence and following the geometric distribution with probability function

$$P(N=n)=pq^n\ ,\ 0<p<1\ ,q=1-p\ ,\ n=0,1,2...$$

We consider the nonnegative geometric random sum

$$L=X_1+X_2+...+X_N$$

The present paper is mainly devoted to the practical applications of the above random sum in the area of information risk control operations.

From the fact that an explicit evaluation of the

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distribution function of the formulated random sum is not possible it is advisable to evaluate the corresponding characteristic function. It is readily demonstrated that the characteristic function of the random sum is given by

$$\varphi_L(u) = \frac{p}{1 - q\varphi_X(u)}$$

The infinite divisibility of the above characteristic function provides very good theoretical and practical reasons for establishing the distribution function of the corresponding geometric random sum of nonnegative random variables as a member of the class of selfdecomposable distribution functions. The inclusion of selfdecomposable distribution functions into the class of unimodal distribution functions makes quite clear the significance of such an establishment. The presence of a unique mode for the distribution function of the proposed geometric random sum extends its applicability as a strong analytical tool in decision making under conditions of risk. In particular the presence of a unique mode at the point zero for such a distribution function makes the corresponding random sum suitable for evaluating risk control operations. It is readily understood that the establishment of the selfdecomposability of the above infinitely divisible characteristic function must be based on the properties of the characteristic function

$$\varphi_X(u)$$

It seems to be of some theoretical and practical importance to consider a well known particular case of the characteristic function of the formulated nonnegative geometric random sum (Prabhu [10]). . The main characteristic of such a particular case is the presence of the exponential distribution in the constituent elements and the form of the proposed geometric random sum. More precisely, we suppose that the nonnegative random variable

$$X$$

follows the exponential distribution with characteristic function

$$\varphi_X(u) = \frac{\nu}{\nu - iu}\ ,\ \nu > 0$$

In this case the characteristic function of the random sum L is given by

$$\varphi_L(u) = p + q\frac{\vartheta}{\vartheta - iu}$$

where

$$\vartheta = p\nu$$

The above particular case of the characteristic function of the random sum

$$L = X_1 + X_2 + ... + X_N$$

is a Bernoulli random sum of exponentially distributed random variables with parameter

$$\vartheta = p\nu$$

The above random sum is a nonnegative analogue of the discrete geometric random sum which has been introduced and investigated by Artikis [11]. The stochastic derivation and the applicability of the geometric random sum of nonnegative, independent and identically distributed random variables are contained in the research purposes of the present paper. The paper also concentrates on the role of such a geometric random sum as a strong analytical tool for the support of learning processes in the area of risk evaluation and risk treatment operations. More precisely, it is shown that the mathematical structure, stochastic derivation and the properties of the proposed geometric random sum are particularly useful for developing learning processes in various significant areas of the information risk management theory and practice.

3. Stochastic Derivation of a Nonnegative Geometric Random Sum

The present section of the paper concentrates on the establishment of a stochastic derivation of the nonnegative geometric random sum proposed by the previous section. We consider the homogeneous Poisson process

$$\{N(t),\ t \geq 0\}$$

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with intensity

$$\lambda$$

and the random variable

$$S$$

following the exponential distribution with parameter

$$\mu$$

It is easily shown that the discrete random variable

$$N,$$

denoting the number of the events of the Poisson process in the time interval

$$[0,S],$$

follows the geometric distribution with probability function

$$P\left(N=n\right)=p\,q^{n}\ ,\ n=0,\,1,\,2...$$

where

$$p=\frac{\mu}{\mu+\lambda}$$

We also suppose that an event of the homogeneous Poisson process activates an operation of a given type. Moreover, we suppose that the nonnegative random variable

$$X_{n},\ n=1,2,...$$

denotes the duration of the operation of a given type activated by th event of the homogeneous Poisson process in the time interval

$$[0,S]$$

It easily follows that the nonnegative geometric random sum

$$L=X_{1}+X_{2}+...+X_{N}$$

denotes the total duration of operations of a given type activated by

$$N$$

events of the homogeneous Poisson process in the time interval

$$[0,S]$$

Since the homogeneous Poisson process and the exponential distribution are generally recognized as very important tools of stochastic modelling, then it can be said that the stochastic derivation of the nonnegative geometric random sum

$$L=X_{1}+X_{2}+...+X_{N}$$

which is established by the present section of the paper, provides analysts and decision makers with valuable information for investigating practical problems. In addition, the practical applicability of the above random sum is significantly extended by the assumption that the random variables of the sequence

$$\left\{X_{n},\ n=1,\ 2,...\right\}$$

follow an arbitrary distribution. The possibility of selecting the form of the characteristic function

$$\varphi_{X}\left(u\right)$$

has as a direct consequence the determination of a very wide class of nonnegative geometric random sums. The present paper makes clear that the members of such a class are particularly useful for developing learning processes in the area of risk control operations, which are the most challenging and dynamic operations of the risk management process (Bernstein[12], Kloman [13]).

4. Incorporating a Nonnegative Geometric Random Sum in Learning Processes and Information Risk Control

The present section of the paper concentrates on the interpretation of the nonnegative geometric

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random sum as a constituent factor of learning processes for the description, selection and implementation of information risk control operations.

We consider a system facing a risk at the time point 0, being the beginning of its activity, and we suppose that the random variable

$$N(t)$$

of the homogeneous Poisson process

$$\{N(t),\ t \geq 0\}$$

denotes the frequency of the risk in the time interval

$$[0,\ t]$$

We consider the exponentially distributed random variable

$$S$$

and we suppose that the above random variable denotes the lifetime of the system. It is quite obvious that the geometrically distributed random variable

$$N$$

denotes the frequency of the risk in the time interval

$$[0,S]$$

We also suppose that an occurrence of the risk activates a risk control operation of a given type. Moreover, we suppose that the nonnegative random variable

$$X_n,\ n=1,\ 2,...$$

denotes the duration of the risk control operation of a given type activated by the th occurrence of the risk in the time interval

$$[0,S]$$

It readily follows that the nonnegative geometric random sum

$$L = X_1 + X_2 + ... + X_N$$

denotes the total duration of risk control operations of a given type activated by

$$N$$

occurrences of the risk in the time interval

$$[0,S]$$

The interpretation of the proposed nonnegative geometric random sum as a constituent factor of learning processes can be done in the following way. The stochastic derivation of the nonnegative geometric random sum

$$L = X_1 + X_2 + ... + X_N$$

is based on a homogeneous Poisson process

$$\{N(t),\ t \geq 0\}$$

the exponentially distributed random variable

$$S$$

and the sequence

$$\{X_n,\ n=1,\ 2,...\}$$

which are fundamental stochastic models. Since learning processes are partially stochastic modelling processes where various stochastic models are developed in establishing an understanding of reality, then it easily follows that the proposed nonnegative geometric random sum can be interpreted as a constituent factor of learning processes in the discipline of risk management.

The consideration of the proposed nonnegative geometric random sum as a strong analytical tool for managing risks threatening an information system seems to be of particular practical importance. The term information system means a system of persons, data records and activities that handle the data and information in a given organization, including manual or automated processes. In addition, the term information risk means a risk of which the cause can damage an information system. Information risk

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management can be defined as the systematic process of managing the information risks of an organization in order to implement its objectives in a way consistent with public interest, human safety, environmental factors and the law. It consists of the planning, organizing, directing and controlling activities undertaken with the goal of developing an efficient information risk management program that reduces the adverse impact of information risks on an organization. In the last ten years risk managers have responded actively and creatively to information risks. The very strong connections of modern organizations with information systems make imperative a remark on the great importance of stochastic models for the description, analysis and implementation of information risk management operations. It is generally recognized that information risk causes such a fire, flood, earthquake, electrical and magnetic disturbances, change in temperature and humidity, hardware and software failure, human error and criminal action continuously threaten the proper function of an information system. Such risk causes can give rise to an information system breakdown, which subsequently can have a severe impact on the continuity of the activities of an organization. It is easily understood when it comes to protecting information systems from the adverse effects of information risk causes, risk managers should make use of information risk control and information risk financing operations. For protection of information systems, information risk control operations constitute the first preference and information risk financing operations come second (Artikis et al [14]). Empirical investigations concerning the risk management practices of modern organizations have made quite clear that risk managers have started to handle information risks by considering information risk control operations as very significant elements of information risk management programs. From the findings of these studies, we can get the conclusion that stochastic models are particularly useful for assessing, selecting and implementing information risk control operations (Wright [7] ,Kloman [13]). The geometric random sum of nonnegative random variables, interpreted as the total duration of applying a risk control operation to a geometrically distributed number of occurrences of a given information risk, can provide risk managers with useful information for protecting the information system of a modern complex organization. It is quite obvious that such a protection can be implemented by making use of the probabilistic behaviour of the proposed geometric random sum of nonnegative random variables. This means that the investigation of the theoretical properties of such a random sum can extend its practical applicability to information risk management and other fundamental areas of information management.

The proposed interpretation of the nonnegative geometric random sum

$$L = X_1 + X_2 + ... + X_N$$

as a constituent factor of learning processes and the acceptance of an information system as a service system support the consideration of such a random sum as a strong analytical tool for investigating the performance of information risk control operations, which constitute particularly valuable operations of the modern information risk management process. It seems that the above random sum can contribute to the development and implementation of efficient information risk management programs.

5. Conclusion

The consideration and stochastic derivation of the geometric random sum of nonnegative, independent and identically distributed random variables constitute the theoretical contribution of the paper. Moreover, the practical contribution of the paper includes the interpretation of the formulated nonnegative geometric random sum as a constituent factor of learning processes, useful for supporting control operations applied to different types of information risk.

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